

**INVESTIGATIONS AND IMPROVEMENTS IN MICROWAVE OPTO-ELECTRONIC  
VARIABLE DELAY LINES**

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**Abstract**

A particular class of optically generated microwave delay lines is dealt with namely those that apply path length dispersions. Detailed analysis of the design, using an acousto-optic device for spatial imaging is given. Furthermore two new designs, decomposing the signal into sub-bands; a.) via a pixelled mirror and b.) using an electrical filter bank, are discussed. Detailed analysis of these structures as well as limitations of applicability are given. Numerical experiments validate the theoretical results.

**1. Introduction**

Among their numerous applications, variable delay lines comprise a key element of microwave phased array antennas. They are needed in order to resolve the so called *beam squinting* problem, occurring in wide band phased arrays, where the radiating elements are fed by variable phase shifters. There are two methods to resolve the problem of erroneous detection of the angular spectrum caused by beam squinting. One can either recalculate the true angular spectrum from the knowledge of the exact phase shift introduced at any RF frequency, or one can compensate for this effect a-priori by applying frequency-proportional phase shifts, i.e. delay lines. These delay lines can be realized either in the RF band, or, more advantageously, in the optical band. This work addresses the latter method.

A number of realizations of optical delay lines, both of direct detection and of the heterodyning design, have been disclosed in the literature; the principles of much of these are summarized in [1]. One of the most promising concepts, introduced in [2] is based on the concept of path-length dispersion, where the RF spectrum is spatially decomposed and each frequency component delayed, using one pixel of a *Deformable Mirror Device* (DMD) [3], by the optical path-length travelled: the structure is dispersive, i.e. this path-length is frequency-dependent (in practice it depends linearly on frequency). Optical delay lines realized by path-length dispersion result in structures whose length is commensurate with the optical wavelength [1] rather than being in the order of the antenna dimensions, what is characteristic to more conventional designs.

The first purpose of this paper is to analyse a modified version of this structure, (called, in what follows a *AOBC-single mirror structure*) where the diffraction grating is re-

placed by an acousto-optic Bragg-cell to realize the spatial imaging of the frequency spectrum. This modification is needed because a detailed analysis of the configuration introduced in [2] has shown that a grating cannot provide sufficient spatial dispersion of the relatively very narrow frequency range resulting from the modulation of an optical carrier with an RF/microwave signal. (Note that this was used in the *experiments* of Ref. [2].) One of the results of our analysis is that this structure causes an unacceptably large linear distortion. In what follows two new structures are introduced being free of this problem. An analysis of these structures is given and their field of application discussed. The organization of the paper is as follows. In Section 2 the notion of group delay realized via optical path length dispersion, is introduced. Section 3 contains an analysis of the modified structure of the original configuration of [1], as mentioned above. The new structures are introduced in Section 4, where it is shown that in order to avoid the delay distortion—i.e. the frequency dependence of the group delay—the optical beam must be divided into sub-bands. This is done in one structure by optical means and by a filter bank in the other. Limiting factors as well as design formulae are presented in Section 5. In Section 6 some conclusions are drawn.

**2. The concept of path-length dispersion**

As it is well known [1, 4] in wide-band phased arrays a progressive time delay rather than a progressive phase shift has to be applied in the excitation of a radiating element located at  $y$ . To point the beam in direction  $\vartheta$ , delay of

$$\tau = \frac{y}{c} \cdot \sin \vartheta \quad (1)$$

has to be applied.

This has to be realized in the frequency band of the signal, i.e.  $\omega \in (\omega_L, \omega_U)$  with  $L$  and  $U$  meaning lower and upper frequency limits. In phased array literature this is called true time delay (TTD). In one possible realization of TTD the signal is transmitted through a transmission line or free space section of length

$$x = \frac{v}{c} y \sin \vartheta \quad (2)$$

In a different realization the RF signal is converted to the optical band, this signal is spectrally decomposed and each frequency component transmitted through dispersive paths. Each path travelled is proportional to frequency difference  $\Delta\omega = \omega - \omega_L$ . Optical path length is thus

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$$\xi + \Delta\xi = \xi + \eta \frac{\Delta\omega}{\omega_u - \omega_L} \quad (3)$$

As shown in [1]

$$\xi = \varphi_L \frac{c}{\omega_{oL}}; \frac{\eta}{\xi} = D - 1; D = \frac{\omega_u}{\omega_L} \cdot \frac{\omega_{oL}}{\omega_{oU}} \quad (4)$$

where subscript  $o$  designates optical frequency. Note that  $x$  is in the order of the RF wavelength while  $\xi$  is in the order of the optical wavelength.

### 3. Analysis of the AOBC-single mirror structure

The configuration of an acousto-optic Bragg cell-single mirror configuration is illustrated in Fig. 2. It consists of a Bragg-cell on the left, where the RF/microwave signal modulates the optical carrier. The Bragg cell associates an angle to each spectral component of the RF/microwave signal and the resulting beam is

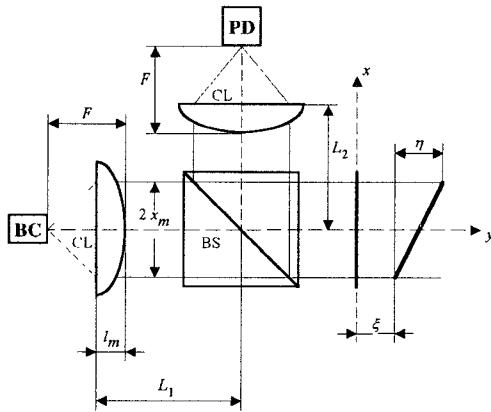


Fig. 1 The AOBC-single mirror structure

subsequently collimated by a cylindrical lens. The collimated beam travels through a beam splitter and is reflected by a mirror—i.e. by one pixel of the DMD. Both the location  $\xi$  of the mirror and its angle are adjustable (note that frequency dependent path-length is accomplished by this rotation). Finally, the reflected beam is redirected by the beamsplitter into a second cylindrical lens which focuses the beam onto the photodiode.

It can be shown that the total phase shift from the Bragg cell to the photodiode is

$$\varphi(\Omega) = \varphi_0 (\Omega + 1) \left[ q + \xi + \frac{1}{\sqrt{1 - \frac{\Omega^2}{a^2}}} - \frac{\Omega^2}{1 - \frac{\Omega^2}{a^2}} \frac{s}{u^2 a^2} \right] \quad (5)$$

where  $\varphi_0$  is twice the optical path delay over the focal length  $F$ ,  $a$  is a parameter which is determined by the refractive index of the acoustic velocity in the Bragg cell,  $u = x_m/F$ ,  $s = (n-1)l_m/F$  ( $n$  is the refractive index of the lens),  $q$  includes all fixed lengths, normalized with respect to  $F$ ,  $\xi$  is

the variable position of the mirror in terms of the relative modulation frequency

$$\Omega(x) = \frac{\omega_o - \omega_{oL}}{\omega_{oL}} = a \frac{ux}{\sqrt{1 + u^2 x^2}} \quad (6)$$

As indicated by (6), the fan-out of the optical beam in the focal regions and the parabolic profile of the cylindrical lenses result in a spatially and, by implication, a frequency dependent phase shift through the device. As this frequency dependence is not linear, it causes group-delay distortion what should be compensated.

### 4. The discretized structure

In order to compensate this distortion, a mirror of curved surface rather than a plane mirror has to be applied for each pixel of the DMD. This is approximated by a structure in which the mirrors are realized in the form of pixels, approximating the desired surface; the required delay variation is accomplished by shifting these pixels by  $\Delta\xi$ , according to Eq. (3). Note that in this way the light beam, and consequently the spectrum is subdivided into sub-bands. This discretized structure is shown in Fig. 3; position  $I$  of the mirrors corresponds to the reference (0) delay while in position  $II$  there is a certain additional delay.

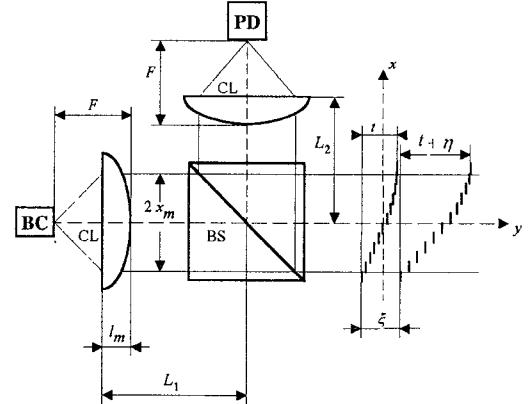


Fig. 2 A discretized variable delay line

To minimize the delay distortion we have chosen the normalized mirror profile to be

$$\xi_0(x) = sx^2 - \sqrt{1 + u^2 x^2} - au(q + \xi)x \quad (7)$$

Using such a profile and selecting a bandwidth of 500 MHz and a focal length of 1 cm, the computed time delay vs. frequency of the device shown in Fig. 3 at  $\xi=0$ . The other parameters used in this numerical experiment were as follows: acoustic velocity 5000 m/s, Bragg cell refractive index 2, optical wavelength 0.633 μm. The profile itself, normalized to  $F$  is plotted in Fig. 4.

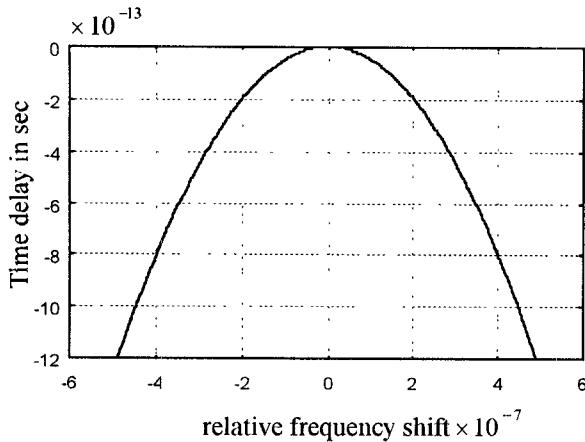


Fig. 3 Computed time delay at  $\xi=0$

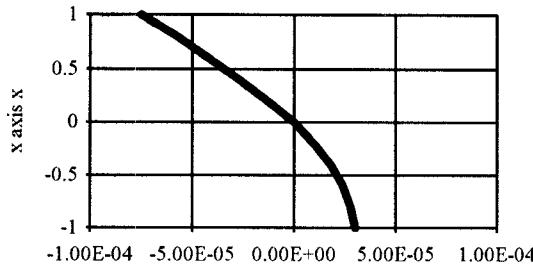


Fig. 4 The normalized mirror profile

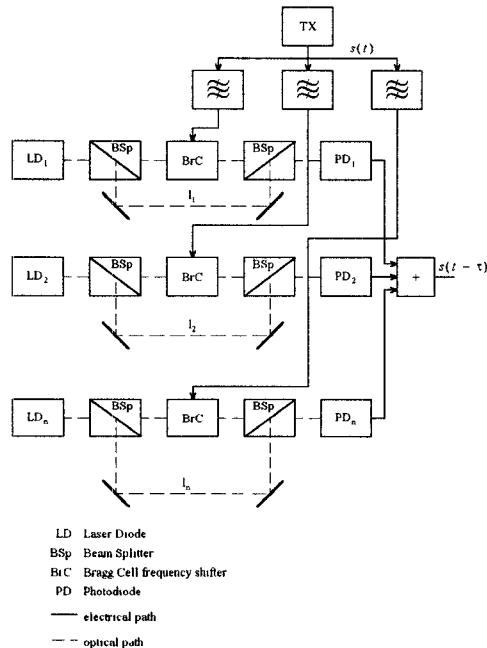


Fig. 5 Delay line using a filter bank

As an alternative to the configuration described above an RF filter-bank can be used to discretize the spectrum. The block-diagram of this concept is shown in Fig. 5.

It can be seen that in this realization the filters play the same role as the pixelling of the mirror in the former configuration.

### 5. Design concepts

Although the two structures introduced in Section 4 are quite different from each other, the operations realized by them are identical. Thus their limitations as well as design concepts of the main parameters are the same. The following discussion is valid for both. In some statements filter characteristics will be mentioned explicitly.

With designations introduced previously sub-band center frequencies are given by

$$\omega_i = \omega_L + \frac{B}{2n}(2i-1); \quad i = 1, 2, \dots, n \quad (8)$$

The bandwidth of the individual filters can be chosen as  $B_i = B/n$ . In phased array applications the phase shift and the group delay, applied to the radiating element at location  $y$  are given in Eq.-s (1), (3) and (4). This phase shift and the group delay can be approximated by applying a discretized optical path-length

$$l_i = y \sin \theta \left[ \frac{\omega_L}{\omega_{oL}} + \left( \frac{\omega_U}{\omega_{oU}} - \frac{\omega_L}{\omega_{oL}} \right) \frac{2i-1}{2n} \right] \quad i=1,2,..,n \quad (9)$$

Using these results the electrical transfer function of the entire device becomes

$$H(\omega) = \sum_{i=1}^n H_i(\omega) \exp \left\{ 2\pi j \left( \omega + \omega_{oL} - \omega_L \right) \frac{y \sin \theta}{c} \times \left[ \frac{\omega_L}{\omega_{oL}} + \left( \frac{\omega_U}{\omega_{oU}} - \frac{\omega_L}{\omega_{oL}} \right) \frac{2i-1}{2n} \right] \right\} \quad (10)$$

where  $H_i(\omega)$  is the transfer function of the  $i$ -the filter or pixel.

One of the main limitations of this configuration is the fact that  $H(\omega)$  has transmission nulls at the crossover frequencies

$$\omega_{ci} = \omega_i + \frac{B}{2n} = \omega_{i+1} - \frac{B}{2n},$$

if the phase-shift difference  $\Delta\phi = \omega_{ci}(l_{i+1} - l_i)/c$  is equal to  $\pi$ . To avoid excess linear distortion one should conservatively choose

$$\Delta\phi \leq 2\pi/3. \quad (11)$$

Another problem that arises when the sub-bandwidths  $B/n$  are too narrow is that the filters require unreasonably high  $Q$  factors.

Condition (11) determines the minimum number of sub-bands. It can be shown that in phased arrays

$$n \geq 1.31\delta \% / BW^0 \quad (12)$$

where  $\delta\%$  means the percentage bandwidth i.e.  $\delta\% = 100B/\omega_0$  with  $\omega_0$  the carrier frequency and  $BW$  the array beam-width in degrees.

If the equality is used in Eq (27) we get for the relative bandwidth of the filters:

$$Q_T = 131 / BW^0 \quad (13)$$

The loaded  $Q$  factors of filter resonators are close to (equal to and lower than)  $Q_T$ .

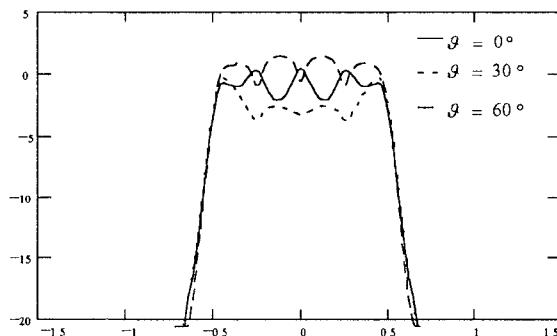


Fig. 6 Transfer function of the analyzed structure

Figures 6 and 7 show computed transfer functions and impulse responses for some delay lines. The main parameters used were: optical carrier 300 THz; electrical bandwidth 214 MHz; array length 10 m; number of sub-bands 4. The filters were of third-order Butterworth type. For the sake of comparison the impulse response of a single band-pass filter of equal  $B$  and equal slope is also shown. It can be seen that a marginal additional distortion is caused only by the new structure.

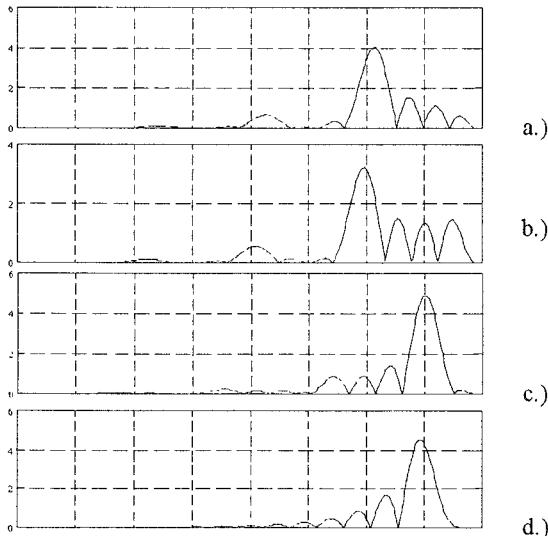


Fig. 7 Impulse responses of the delay line;  
a.)  $\theta=0$ ; b.)  $\theta=30^\circ$ ; c.)  $\theta=60^\circ$ ; d.) single filter

## 6. Conclusions

In this paper TTD via coherent techniques is investigated and particular methods are dealt with by which the concept of path length dispersion comes closer to reality. Note, that the application of this concept decreases the length of the device by several orders of magnitude. Methods are found which decrease the group delay distortion appearing in spectral decomposition via optical means. Two totally different approaches are applied leading to conceptually identical results. In one the distortion is compensated by optical methods, i.e. by modifying the mirror profile and approximating the curved mirror with pixels; thus sub-bands are separately processed, i.e. travelled through different paths. In the other the signal is spectrally decomposed in the RF domain, via microwave filters and then converted to the optical band, where they are (similarly) processed.

Basic limitations are given. It turns out that the number of sub-bands has to be increased if the total bandwidth and/or the antenna dimensions are increased (more precisely concerning the latter: if the antenna beam width is decreased).

The choice between the two configurations depends on particularities: the required number of sub-bands, the required dimension of the DMD, the required  $Q$  factors of the resonators, etc. To handle extremely wide band signals one can foresee a configuration where the two realizations are combined: the RF signal is first decomposed into  $n$  sub-bands using a filter bank followed by spatial decomposition by a stack of Bragg cells. For this device one would still need only one beam splitter and only one cylindrical lens at the input and output ports of the beam splitter, but the required number of photodiodes would have to match the number of Bragg cells and the DMD would now be a two-dimensional array of pixels.

## Acknowledgement

This work was primarily sponsored by OTKA, the Hungarian Foundation for Scientific Research under contract number T 7396. Their support is gratefully acknowledged. One of the authors (O.S.) acknowledges also the support of the National Sciences and Engineering Research Council of Canada

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